

Turbulence induced additional deceleration in relativistic shock wave propagation: implications for gamma-ray burst

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Abstract The late afterglow of gamma-ray burst is believed to be due to progressive deceleration of the forward shock wave driven by the gamma-ray burst ejecta propagating in the interstellar medium. We study the dynamic effect of interstellar turbulence on shock wave propagation. It is shown that the shock wave decelerates more quickly than previously assumed without the turbulence. As an observational consequence, an earlier jet break will appear in the light curve of the forward shock wave. The scatter of the jet-corrected energy release for gamma-ray burst, inferred from the jet-break, may be partly due to the physical uncertainties in the turbulence/shock wave interaction. This uncertainties also exist in two shell collisions in the well-known internal shock model proposed for gamma-ray burst prompt emission. The large scatters of known luminosity relations of gamma-ray burst may be intrinsic and thus gamma-ray burst is not a good standard candle. We also discuss the other implications.

Keywords gamma-ray burst; turbulence; shock wave

1 Introduction

Gamma-ray burst (GRB) is the most explosive event in the universe. The standard picture for GRB is the relativistic fireball shock model (Paczynski 1986; Goodman 1986; Shemi & Piran 1990; Rees & Mészáros 1992; Mészáros & Rees 1993; Rees & Mészáros 1994). Within such a picture, an initially hot fireball composed of photons, electron-positron pairs, and a small amount of baryons expands outward because of the large optical depth, converts most of its thermal energy into the

bulk kinetic energy of the baryons to form a relativistic cold shell; the expanding shells interact with each other and with the surrounding medium, causing their kinetic energy to be radiated in shock waves and producing the observed GRB prompt and afterglow emissions. Within such a scenario, the relativistic shock generates the magnetic field via Weibel instability (Weibel 1959; Medvedev 1999) and the energetic electrons via first order Fermi acceleration which cool down, most likely via synchrotron emission (Mészáros et al. 1994; Tavani 1996). Although the standard fireball model can explain the general features of GRB: the early-time rapid temporal variability and late-time smooth afterglow, there are some observational features beyond the expectations of this model such as the early X-ray plateaus, various rebrightenings and chromatic breaks (Zhang 2011). Different extensions of the basic model are invoked to explain the observed deviation from the model. The extensions include the modification of the total energy of the ejecta, the environment, the microphysics parameters and the radiative mechanism (Zhang 2011). However, often these are tailored on a burst by burst basis.

Most astrophysical system, e.g., accretion disks, solar/stellar winds, and the interstellar medium (ISM) are in turbulent states with embedded magnetic fields that influence almost all of their properties (Biskamp 2005; Frisch 1995; Goldstein et al. 1995; elmegreen & Scalo 2004; Scalo & Elmegreen 2004). Narayan & Kumar (2009) (Narayan & Kumar 2009) and Lazar et al. (2009) (Lazar et al. 2009) have proposed a relativistic turbulence model instead of the internal shock model as the production mechanism for fast variable GRB light curves and applied it to GRB 080319B (Kumar & Narayan 2009). Zhang & Yan (Zhang & Yan 2011) have also developed a new model of GRB prompt emission in the highly magnetized regime, namely, the Internal-Collision-induced Mag-

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netic Reconnection and Turbulence model which not only carries the merits, but also alleviate some drawbacks of the internal shock model. The role that the magnetic fields play in acceleration, collimation and emission production in GRB outflows remains one of the central issues. As an alternative to microscopic Weibel instability, a production mechanism of magnetic field by macroscopic turbulence is proposed (Goodman & MacFadyen 2007; Sironi & Goodman 2007) and verified by recent three-dimensional relativistic magnetohydrodynamics simulation (Zhang et al. 2009). This simulation also show that the macroscopic turbulence produces small-scale moving magnetic clouds which is likely a site for Fermi acceleration of charged particles. It is possible that the turbulence may give a self-consistent picture of GRB emission, which includes the fast variability, the acceleration of nonthermal electrons and the amplification of magnetic fields.

The pre-shock turbulence can be triggered by the interaction between GRB precursors and the interstellar medium through fluid instabilities. Cosmic rays acceleration in relativistic collisionless shock can also excite large-scale turbulence in the shock upstream (Milosavljević & Nakar 2006). In this paper, we focus on how the turbulence/shock wave interaction modify the propagation of the relativistic shock. Non-relativistic shock wave propagation in turbulent interplanetary plasma had been studied, the result of which show that the solar wind plasma turbulence may put considerable contribution in the shock wave deceleration (Chashei & Shishov 1996). In the fireball shock model, the interaction of ultra-relativistic ejecta with the surrounding medium drives a relativistic shock which satisfies the Blandford-McKee (BM) self-similar solution and totally determines the time evolution of the afterglow emission (Blandford & McKee 1976). If the turbulence exists in the upstream medium, we show that the relativistic shock will also amplify the turbulence behind the moving shock front, and thus decelerates more quickly than that BM solution predicted. We derive below the evolution solution of the relativistic shock propagating in a turbulent medium and discuss the implications for GRBs.

2 Turbulence induced additional deceleration of relativistic shock

We consider a relativistic GRB jet sweeping up the surrounding medium produced by the compact GRB progenitor with density $\rho = Ar^{-s}$, where $A = nm_p$ for interstellar medium (ISM) environment while $A = 5 \times 10^{11} A_*$ g cm⁻³ for wind environment (Chevalier & Li 2000). The hydrodynamics involves a relativistic blast wave expanding into the wind. For an ultrarelativistic, adiabatic blast wave, Blandford & McKee find that

$$E = \frac{8\pi A \Gamma^2 R^{3-s} c^2}{17 - 4s}, \quad (1)$$

where E is the isotropic energy of the blast wave (mostly denoted by E_{iso}) and R is the shock wave radius. In the observer's frame, there is $t = R/(2\Gamma^2 c)$ and thus $\Gamma \propto t^{(s-3)/(8-2s)}$.

The turbulence in the wind will make Γ decay more faster. We first review the basic idea of energy transformation process from non-relativistic shock wave to the turbulence, then generalize it to the relativistic shock. If the turbulence in the regions before and behind the shock surface is of acoustic type and it induced sound wave is incident normally from the front on a non-relativistic shock wave, the turbulence energy transformation coefficient is

$$\chi = \frac{W_d}{W_u} = \eta^2 \left[\frac{2\gamma}{\gamma + 1} \right] M_u^2, \quad (2)$$

where W_d and W_u are the energy density of turbulence in the downstream and upstream of the shock, M_u is the upstream Mach number, γ is the ratio of specific heats, and $\eta \simeq 1/(\gamma + \sqrt{2\gamma(\gamma - 1)})$. For $\gamma = 5/3$, we have $\chi \simeq 0.1 M_u^2$ which means the considerable amplification of turbulence for strong shock waves $M_u \gg 1$. The source of the turbulence amplification is the shock wave energy. Chashei & Shishov showed that the relative level of turbulence $\delta_d = W_d/p_d = \eta^2 \delta_u$ is small, which means that the turbulence behind the shock is always weak (Chashei & Shishov 1996).

For a relativistic shock, the Rankine-Hugoniot relations with the turbulence taking into account have the following form

$$\begin{aligned} \Gamma_u n_u \beta_u &= \Gamma_d n_d \beta_d \\ w_u \Gamma_u^2 \beta_u^2 + p_u + \pi_u &= w_d \Gamma_d^2 \beta_d^2 + p_d + \pi_d \\ w_u \Gamma_u^2 \beta_u + H_{\text{tu}} &= w_d \Gamma_d^2 \beta_d + H_{\text{td}}, \end{aligned} \quad (3)$$

where $\pi_{u,d}$ are the turbulence momentum fluxes and $H_{\text{tu},\text{td}}$ are the turbulence energy fluxes. The relation between $\pi_d(H_{\text{td}})$ and $\pi_u(H_{\text{tu}})$ for relativistic shock is unclear. We just know that the energy flux can be approximated as $H \simeq \Gamma^2 \varepsilon c$, where ε is the energy density of the turbulence in the comoving frame for relativistic flow. It is convenient to define a relative turbulence energy transformation coefficient $\delta_{\text{du}} \equiv (H_{\text{td}} - H_{\text{tu}})/e'$ for relativistic shock, where e' is the energy density of the post-shock medium in the comoving frame. Due to the energy transformation from the

shock to the turbulence, the shock flow total energy decrease. We should adopt an energy equation in the form (Chashei & Shishov 1996)

$$\frac{\partial e}{\partial t} + \nabla \cdot \mathbf{H} = 0, \quad (4)$$

where e is the energy density of the fireball (without the turbulence energy) in the central engine frame, \mathbf{H} is the turbulence energy flux. If the turbulent terms are neglected in (3), the BM solution is achieved. If the turbulence is included, the BM solution is not valid any more.

We shall use the perturbation method to solve (4) under the condition $\delta_{\text{du}} \ll 1$. In the central engine frame, we have $dR = v_s dt$, where v_s is the velocity of the shock wave. Carrying out the volume integration of the energy equation with including the energy losses (caused by turbulence) on the shock surface, we get

$$v_s \frac{d}{dR} \left(\int 4\pi r^2 e dr \right) = -(H_{\text{td}} - H_{\text{tu}}) 4\pi R^2. \quad (5)$$

Since $\int 4\pi r^2 e dr = E$ for zero approximation, and the shock is relativistic $v_s \sim c$, the above solution can be written in the form

$$\frac{dE}{dR} \approx -4\pi \delta_{\text{du}} e' R^2. \quad (6)$$

In principle, δ_{du} depends on the detailed process of the turbulence/shock interaction. To obtain an analytical solution, we assume $\delta_{\text{du}} = \text{const}$.

ISM environment ($s = 0$). The post-shock energy density is $e' \simeq 4\Gamma^2 n m_p c^2$ and the total energy of the fireball is $E = 8\pi n m_p \Gamma^2 R^3 c^2 / 17$. Then the solution of (6) is

$$\Gamma \propto R^{-\frac{3}{2}} \exp \left(-17 \int_{R_0}^R \frac{\delta_{\text{du}}}{r} dr \right), \quad (7)$$

where R_0 is the initial distance of the shock occurs. For the case $\delta_{\text{du}} = \text{const}$, we have $\Gamma \propto R^{-3/2-17\delta_{\text{du}}}$. Using the relation $R = 2\Gamma^2 ct$, the evolution solution of relativistic shock is $\Gamma \propto t^{-3/8-\delta_{\text{du}}/(16/17+\delta_{\text{du}}/8)} \equiv t^{-3/8-f_{\text{dec}}}$.

WIND environment ($s = 2$). The post-shock energy density is $e' = 4\Gamma^2 A R^{-2} c^2$ and the total energy is $E = 8\pi A \Gamma^2 R c^2 / 9$. We get $\Gamma \propto R^{-1/2-9\delta_{\text{du}}}$ and thus the evolution solution is $\Gamma \propto t^{-1/4-\delta_{\text{du}}/(4/9+4\delta_{\text{du}})} \equiv t^{-1/4-f_{\text{dec}}}$.

Additional deceleration factors are involved as compared with the BM solutions for ISM and wind environment respectively. We note that the coefficient δ_{du} in fact includes the energy transformation process between the shock wave and the turbulence, which may be in principle defined more precisely. By defining it, our results can also be applicable for MHD turbulence.

3 Implications and discussions

In this work, we have demonstrated that the relativistic blast wave involving the turbulence in the upstream decelerates more quickly than what BM solution predicted based on the condition that the relative turbulence energy transformation is small. This dynamical effect changes temporal behavior of the peak frequencies and thus the slope of the light curves. We restrain our discussions for ISM case. The position of the peak of the spectrum F_ν varies as $\nu_c \propto t^{-1/2+4f_{\text{dec}}}$ in fast cooling while $\nu_m \propto t^{-3/2-4f_{\text{dec}}}$ in slow cooling. The main results are summarized in Tab. 1. Emission for a forward shock predicts definite relation between the spectral and temporal indexes, $F_\nu \propto t^{-\alpha} \nu^{-\beta}$ (Rees & Mészáros 1998). Observed afterglow typically do not comply with this prediction (Racusin 2009). The deviation of the observed closure relation from the external shock afterglow model in those bursts such as GRB050413, GRB0607A and GRB061202 may be caused by the upstream turbulence (Liang et al. 2007).

The outflows of GRB are believed to be ultra-relativistic jets. Since the outflow decelerates more quickly, a jet-break in afterglow will appear earlier when $\Gamma \sim 1/\theta_j$, where θ_j is the jet opening angle. Using $\Gamma \propto R^{-3/2-17\delta_{\text{du}}}$ and $\Gamma_0^2 R_0^3 = 17E_{\text{iso}}/8\pi n m_p c^2$ (Γ_0 is the initial Lorentz factor of the jet), we get the jet-break time

$$t_j \sim 1.4 \left(\frac{E_{\text{iso},53}}{n_0} \right)^{\frac{1}{3}} \theta_{j,-1}^{\frac{8}{3}} (\Gamma_0 \theta_j)^{-\frac{68\delta_{\text{du}}}{9+102\delta_{\text{du}}}} \text{ days}, \quad (8)$$

where $\theta_j = 0.1\theta_{j,-1}$, $E_{\text{iso}} = 10^{53} E_{\text{iso},53}$ erg and $n = 1n_0 \text{ cm}^{-3}$. (For a GRB located at redshift z , the observed time should be increased by a factor $1+z$.) The jet-break time is earlier by a factor of the last term in (8) than that of the adiabatic blast wave predicted. For $\Gamma_0 = 1000$, $\theta_j = 0.1$ and $\delta_{\text{du}} = 0.1$, the value of the factor is about 0.2 which means that the previous jet opening angle is notably overestimated. The jet-corrected energy $E_\gamma \simeq \frac{1}{2}\theta_j^2 E_{\text{iso}}$ should be potentially re-evaluated by (8). If the turbulence/shock interaction is universal, i.e., the same value of δ_{du} for all GRBs, the three parameters $(t_j, E_{\text{iso}}, \Gamma_0)$ should be statistically correlated. Once obtain the correlation, we can evaluate θ_j and E_γ . While it's not easy to identify the jet-break (Liang et al. 2008; Racusin 2009). Because in most GRBs, the optical and X-ray breaks are chromatic which is contradict with the forward shock model. A natural but not affirmative way to remove the contradiction is to invoke a different origin of X-ray emission: dust scattering (Shao et al. 2007) or photosphere emission of the engine activity (Wu & Zhang 2011). So the late optical observation is crucial to identify the

jet-break. Secondly, the detection of very early optical afterglow peak can provide a direct measurement of the initial Lorentz factor (Vergani 2007). There are many optical afterglows without detailed X-ray coverage before *Swift* era. Now, that *Swift*-XRT provides impressive X-ray light curves, there are too few optical light curves. The Chinese-French mission SVOM is a multi-wavelength GRB observatory scheduled to launch in 2014-2015 (Paul et al. 2011). Its operation window overlapping with that of *Fermi* and *Swift* will shade light on this problem.

The turbulence/shock interaction make some shock energy stored in turbulent state, which lead to some uncertainties and differences between the observational and real values of t_j , E_{iso} , L_{iso} (isotropic luminosity) and E_{peak} (peak energy of νF_ν spectrum). A number of relations involving these parameters were proposed (see, e.g., (Frail et al. 2001; Amati et al. 2002; Ghirlanda et al. 2004; Yonetoku et al. 2004; Liang & Zhang 2005)), which lead to identification of a “GRB standard candle” for cosmology information complementing that derived from SNe. This requires that the energy and the luminosity are precisely estimated from observational quantities. But the significant dispersions of these correlations prevent GRB as a good standard candle, which may be intrinsic due to the lack of the full knowledge of the turbulence/shock interaction.

Those quantities related to the shock will be effected by the turbulence, so do the correlations between these quantities. Although current understanding of the turbulence/shock interaction is limited, posing a challenge to accurate prediction of this highly non-linear phenomenon, the results show the potential ability of the turbulence to solve some problems in GRBs. The results of this letter are also applicable for SNe, AGN and microquasar.

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Table 1 Temporal index α and spectral index β in afterglow for ISM case, the convention $F_\nu \propto t^{-\alpha}\nu^{-\beta}$ is adopted. The power law index of the electron distribution is $p > 2$.

	α	β	$\alpha(\beta)$
slow cooling			
$\nu < \nu_m$	$-\frac{1}{2} - \frac{4}{3}f_{\text{dec}}$	$-\frac{1}{3}$	$\alpha = \frac{3}{2}\beta + 4f_{\text{dec}}\beta$
$\nu_m < \nu < \nu_c$	$\frac{3(p-1)}{4} + 2f_{\text{dec}}(p-1)$	$\frac{p-1}{2}$	$\alpha = \frac{3}{2}\beta + 4f_{\text{dec}}\beta$
$\nu_c < \nu$	$\frac{3p-2}{4} + 2f_{\text{dec}}(p-2)$	$\frac{p}{2}$	$\alpha = \frac{3\beta-1}{2} + \frac{4f_{\text{dec}}(p-2)}{p}\beta$
fast cooling			
$\nu < \nu_c$	$-\frac{1}{6} + \frac{4}{3}f_{\text{dec}}$	$-\frac{1}{3}$	$\alpha = \frac{\beta}{2} - 4f_{\text{dec}}\beta$
$\nu_m < \nu < \nu_c$	$\frac{1}{4} - 2f_{\text{dec}}$	$\frac{1}{2}$	$\alpha = \frac{\beta}{2} - 4f_{\text{dec}}\beta$
$\nu_c < \nu$	$\frac{3p-2}{4} + 2f_{\text{dec}}(p-2)$	$\frac{p}{2}$	$\alpha = \frac{3\beta-1}{2} + \frac{4f_{\text{dec}}(p-2)}{p}\beta$

References

- B, Paczyński, *Astrophys. J.* **308**, L43, 1986
- J. Goodman, *Astrophys. J.* **308**, L47 (1986)
- A. Shemi and T. Piran, *Astrophys. J.* **365**, L55 (1990)
- M. J. Rees and P. Mészáros, *Mon. Not. R. Astron. Soc.* **258**, 41 (1992)
- P. Mészáros and M. J. Rees, *Astrophys. J.* **405**, 278 (1993)
- M. J. Rees and P. Mészáros, *Astrophys. J.* **403**, L93, (1994)
- E. S. Weible, *Phys. Rev. Lett.* **2**, 83, (1959)
- M. V. Medvedec and A. Loeb, *Astrophys. J.* **526**, 697, (1999)
- P. Mészáros *et al.*, *Astrophys. J.* **432**, 181, (1994)
- M. Tavani, *Astrophys. J.* **466**, 768, (1996)
- Bing Zhang, *C. R. Physique.* **12**, 206, (2011)
- D. Biskamp, *Magnetohydrodynamic Turbulence* (Cambridge University Press, UK, 2005)
- U. Frisch, *Turbulence. The Legacy of A. N. Kolmogorov* (Cambridge University Press, UK, 1995)
- M. L. Goldstein, *et al.*, *Ann. Rev. Astron. Astrophys.* **33**, 283, (1995)
- B. G. Elmegreen and J. Scalo, *Ann. Rev. Astron. Astrophys.* **42**, 211, (2004)
- J. Scalo and B. G. Elmegreen, *Ann. Rev. Astron. Astrophys.* **42**, 275, (2004)
- R. Narayan and P. Kumar, *Mon. Not. R. Astron. Soc.* **394**, L117, (2009)
- A. Lazar *et al.*, *Astrophys. J.* **695**, L10, (2009)
- P. Kumar and R. Narayan, *Mon. Not. R. Astron. Soc.* **395**, 472, (2009)
- Bing Zhang and Huirong Yan, *Astrophys. J.* **726**, 90, (2011)
- J. Goodman and A. MacFadyen, *J. Fluid. Mech.* **604**, 325, (2007)
- L. Sironi and J. Goodman, *Astrophys. J.* **671**, 1858, (2007)
- Weiqun Zhang *et al.*, *Astrophys. J.* **692**, 40, (2009)
- I. V. Chashei and V. I. Shishov, *Astrophysics. And. Space. Science.* **243**, 23, (1996)
- R. D. Blandford and C. F. McKee, *Physics of Fluids*, **19**, 1130, (1976)
- R. A. Chevalier and Z. Y. Li, *Astrophys. J.* **536**, 195, (2000)
- M. J. Rees and P. Mészáros, *Astrophys. J.* **496**, L1 (1998)
- J. L. Racusin *et al.*, *Astrophys. J.* **698**, 43 (2009)
- E. W. Liang *et al.*, *Astrophys. J.* **670**, 565, (2007)
- L. Shao *et al.*, *Astrophys. J.* **633**, 1319 (2007)
- Xue-Feng Wu, Bing Zhang, submitted
- J. Paul *et al.*, *C. R. Physique.* **12**, 298 (2011)
- S. D. Vergani, *Astrophys. Spa. Sci.* **311**, 197, (2007)
- D. A. Frail *et al.*, *Astrophys. J.* **562**, L55, (2001)
- L. Amati *et al.*, *Astronomy. And. Astrophysics.* **390**, 81, (2002)
- G. Ghirlanda *et al.*, *Astrophys. J.* **616**, 331, (2004)
- D. Yonetoku *et al.*, *Astrophys. J.* **609**, 935, (2004)
- E. W. Liang and Bing Zhang, *Astrophys. J.* **633**, 611, (2005)
- E. W. Liang *et al.*, *Astrophys. J.* **675**, 528, (2008)
- M. Milosavljević and E. Nakar, *Astrophys. J.* **651**, 979, (2006)